Research on Periodic and chaotic oscillations of Laminated Composite Piezoelectric Rectangular Plates with one-to-two internal resonance

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Abstract: The chaotic dynamics of parametrically excited, simply supported laminated composite piezoelectric rectangular plates are analyzed, which are forced by transverse loads. It is assumed that different layers are perfectly bonded to each other with piezoelectric actuator patches embedded in it. The formulas of the laminated composite piezoelectric rectangular plates are derived by von Karman-type equation, third-order shear deformation laminate theory of Reddy, piezoelectric parametric loads and parametric loads acting in both directions x and y are included. The Galerkin approach has been applied to convert partial differential equations to the ordinary differential equations. The method of multiple scales is used to obtain the averaged equations by Maple program. Based on the averaged equations, the chaotic motions of plates are found by numerical simulation. The numerical results show the existence of chaotic motion in averaged equations, the chaotic responses are sensitive to initial conditions especially sensitive to forcing loads and the parametric excitation.

1. Introduction

Laminated composite piezoelectric rectangular plates have been increasingly applied in all kinds of vibration control devices in recent years [1-10]. Among materials used in devices, piezoelectric materials have received increased attention. The plate theory, chosen for laminated composite piezoelectric rectangular plates, is important in the analysis. The three kinds of plate theories are based on the theories Kirchhoff, Midlin and Reddy. The classical laminated plate theory is based on Kirchhoff assumption, which assumed that the plane cross sections initially normal to the plate middle plane before deformation remain plane and normal to that surface during deformation [11]. This is the result of neglecting the transverse shear strains. The Midlin theory, which
is known as the first-order shear deformation theory, defines the displacement field as linear variations of middle plane displacements using shear correction factors. The third-order shear deformation laminate theory of Reddy allows not only for the transverse shear strains, but also for parabolic variations in the strains across the plate thickness, and thus there is no need to use shear correction coefficients in computing the shear stresses \[12\]. We choose the third-order shear deformation laminate theory of Reddy to describe our model.

When come to the nonlinear and chaotic analysis, Hadian and Nayfeh [13] used the method of multiple scales to analyze asymmetric responses of nonlinear clamped circular plates subjected to harmonic excitations and considered the case of a combination-type internal resonance. Pai and Nayfeh [14] presented a general nonlinear theory for the studies on dynamics of elastic composite plates undergoing moderate-rotation oscillations by considering the geometric nonlinearities. Sassi and Ostiguy [15] investigated effects of initial geometric imperfections on the interaction between forced and parametric oscillations for simply supported rectangular plates. Nayfeh and Vakakis [16] used the method of multiple scales to study the subharmonic travelling waves of thin, axisymmetric, geometrically non-linear circular plates and found the non-linear interactions of pairs of modes with coincident linearized natural frequencies. W.Zhang used the global perturbation method to study the global bifurcations and chaotic dynamics of a parametrically excited, simply supported rectangular thin plate [17].

This paper is focused on the studies for the chaotic dynamics of the simply supported at the four-edge laminated composite piezoelectric rectangular plates subjected to in-plane and piezoelectric excitation. The case of 1:2 internal resonance and primary parametric resonance is considered. Firstly, based on von Karman-type equation, we utilize the dynamic version of the principle of virtual displacements to obtain the governing equations of laminated composite piezoelectric rectangular plates. Then, Galerkin method is applied directly to convert partial differential equations to the ordinary differential equations \[18\]. Based on the equations obtained above, the method of multiple scales can be used to find the averaged equations of the original non-autonomous system. Finally, numerical method is used to investigate periodic and chaotic motions of the laminated composite piezoelectric plates. The phase portrait, waveform and power spectrum are plotted to show the nonlinear response of the laminated composite piezoelectric plates. From the results of numerical simulation, it is found that there exist the periodic and chaotic motions in the laminated composite piezoelectric plates system under certain conditions. In addition, it is found that the forcing loads and the parametric excitation have significant influence on the nonlinear dynamical behavior. Therefore, it is thought that through changing forcing loads and the parametric excitation of the laminated composite piezoelectric plates, we can control the response from the chaotic motion to a period motion.
2. Formulation

We consider the simply supported at the four-edge rectangular Laminated composite piezoelectric rectangular plates, where the edge lengths are a and b and thickness is h respectively. Plates are subjected to in-plane excitations, out-plane loads, thermal loads and piezoelectric excitations, as shown in Figure 1. When come to composites, we consider laminated fiber-reinforced composites, which are a hybrid class of composites involving both fibrous composites and laminated techniques. Rectangular Laminated composite piezoelectric rectangular plates are considered as regular symmetric cross-ply laminates have, who have n reinforced layers with principal material directions alternatively oriented at 0° and 90° to the laminate coordinate axes. The classical regular symmetric cross-ply laminates, which have odd-layers with equal thickness of all layers, have all the four edges clamped, and satisfy the symmetry requirement by which coupling between bending and extension is eliminated. But we free the displacement of $x$ at the edge of $y=0$, and the displacement of $y$ at the edge of $x=0$, so the membrane stress is smaller and there exist the coupling between bending and extension.

Subject to the simply supported edge boundary condition

\[ x = 0 : \frac{\partial u}{\partial x} = 0 \quad w = 0 \]
\[ x = a : u = 0 \quad w = 0 \]
\[ y = 0 : \frac{\partial v}{\partial y} = 0 \quad w = 0 \]
\[ y = b : v = 0 \quad w = 0 \]

Fig. 1. The laminated composite piezoelectric rectangular plates

Consider the Reddy third order displace field
\[ u(x, y, z, t) = u(x, y, t) + z \phi_x(x, y, t) - z^2 \frac{4}{3h^2} \left( \phi_z + \frac{\partial w_z}{\partial x} \right), \]

\[ v(x, y, z, t) = v(x, y, t) + z \phi_y(x, y, t) - z^2 \frac{4}{3h^2} \left( \phi_z + \frac{\partial w_z}{\partial y} \right), \]

\[ w(x, y, z, t) = w_z(x, y, t). \quad (1) \]

Where \((u, v, w)\) are the displacement components along the \((x, y, z)\) coordinate directions, \((u_z, v_z, w_z)\) is the deflection of a point on the middle plane \((z = 0)\). \((\phi_x, \phi_y)\) denotes rotations about the \(y\) and \(x\) axes, respectively.

The nonlinear strain-displacement relations
\[ \varepsilon_{x} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2, \quad \varepsilon_{y} = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2, \quad \varepsilon_{z} = \frac{\partial w}{\partial z} + \frac{1}{2} \left( \frac{\partial w}{\partial z} \right)^2, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \quad \gamma_{yz} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \quad (2) \]

Substitution of displacements (1) into the nonlinear strain-displacement relations in (2) yields the strains.

\[ \begin{bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_{x}^{(0)} \\ \varepsilon_{y}^{(0)} \\ \gamma_{xy}^{(0)} \end{bmatrix} + \begin{bmatrix} \varepsilon_{x}^{(1)} \\ \varepsilon_{y}^{(1)} \\ \gamma_{xy}^{(1)} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{z} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{z}^{(0)} \\ \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z}^{(1)} \\ \gamma_{xz}^{(1)} \\ \gamma_{yz}^{(1)} \end{bmatrix} \quad (3) \]

where
\[ \begin{bmatrix} \varepsilon_{x}^{(0)} \\ \varepsilon_{y}^{(0)} \\ \gamma_{xy}^{(0)} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_z}{\partial x} + \frac{1}{2} \left( \frac{\partial w_z}{\partial x} \right)^2 \\ \frac{\partial v_z}{\partial y} + \frac{1}{2} \left( \frac{\partial w_z}{\partial y} \right)^2 \\ \frac{\partial w_z}{\partial x} + \frac{\partial v_z}{\partial y} + \frac{\partial w_z}{\partial z} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{x}^{(1)} \\ \varepsilon_{y}^{(1)} \\ \gamma_{xy}^{(1)} \end{bmatrix} = \begin{bmatrix} \frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w_z}{\partial x^2} + \frac{2}{2} \frac{\partial^2 w_z}{\partial x \partial y} \\ \frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w_z}{\partial y^2} + \frac{2}{2} \frac{\partial^2 w_z}{\partial x \partial y} \\ \frac{\partial \phi_z}{\partial z} + \frac{\partial^2 w_z}{\partial z^2} \end{bmatrix} \]

\[ \begin{bmatrix} \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{bmatrix} = -c_t \begin{bmatrix} \frac{\partial \phi_x}{\partial y} + \frac{\partial^2 w_z}{\partial x \partial y} + \frac{2}{2} \frac{\partial^2 w_z}{\partial x \partial y} \\ \frac{\partial \phi_z}{\partial y} + \frac{\partial^2 w_z}{\partial x \partial y} + \frac{2}{2} \frac{\partial^2 w_z}{\partial x \partial y} \end{bmatrix}, \quad \begin{bmatrix} \gamma_{xz}^{(1)} \\ \gamma_{yz}^{(1)} \end{bmatrix} = -c_t \begin{bmatrix} \frac{\partial \phi_z}{\partial y} + \frac{\partial^2 w_z}{\partial x \partial y} + \frac{2}{2} \frac{\partial^2 w_z}{\partial x \partial y} \\ \frac{\partial \phi_z}{\partial y} + \frac{\partial^2 w_z}{\partial x \partial y} + \frac{2}{2} \frac{\partial^2 w_z}{\partial x \partial y} \end{bmatrix} \]
\[ c_1 = \frac{4}{3} \text{m}^2, \quad c_2 = 3c_1 \]  

(4)

Stress constitutive relations

\[ \sigma_j = \sigma_{o_j} - e_{o_{j}} E_j - k_j T_j, \quad (i, j, k, l = x, y, z) \]  

(5)

where \( E_j \) is the electric field, \( e_{o_{j}} \) is the piezoelectric moduli. And \( T \) is the temperature increment from a reference state. \( k_x, k_y (k_z = 0) \) are the coefficients of thermal expansion along the \( x, y \) directions.

\[ \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 & 0 & 0 \\ \tau_{1x} & \tau_{1y} & 0 & 0 & 0 \\ \tau_{2x} & \tau_{2y} & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} e_{11} \\ e_{22} \\ \gamma_{1x} \\ \gamma_{2y} \end{bmatrix} \]  

(6)

\[ \begin{bmatrix} Q_{11} \\ Q_{12} \\ Q_{21} \\ Q_{22} \end{bmatrix} = \begin{bmatrix} C^4 & 2C^2S^2 & S^4 & 4C^4S^2 \\ C^4S^2 & C^4 + S^4 & C^2S^2 & -4C^4S^2 \\ S^4 & 2C^4S^2 & C^4 & 4C^4S^2 \\ C^4S^2 - C^2S - CS^3 - 2CS(C^4 - S^4) \end{bmatrix} \begin{bmatrix} Q_{11} \\ Q_{12} \\ Q_{21} \\ Q_{22} \end{bmatrix} \]  

(7)

\[ \begin{bmatrix} Q_{11} \\ Q_{12} \end{bmatrix} = \begin{bmatrix} C^2 & S^2 \\ -CS & CS \end{bmatrix} \begin{bmatrix} Q_{44} \\ Q_{55} \end{bmatrix} \]  

(8)

\[ Q_{11} = Q_{22} = \frac{E}{1 - \nu^2}, \quad Q_{12} = Q_{21} = \frac{\nu E}{1 - \nu^2}, \quad Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1 - \nu)} \]  

(9)

\[ C = \cos \theta, \quad S = \sin \theta \]  

(10)

The following Euler-Lagrange equations are derived using the dynamic version of the principle of virtual displacements \([11] \).

\[ \frac{\partial N_{\alpha x}}{\partial x} + \frac{\partial N_{\alpha y}}{\partial y} = I_\alpha \ddot{u}_\alpha + J_\alpha \ddot{v}_\alpha - c_\alpha \ddot{\psi}_\alpha \]  

(11.a)

\[ \frac{\partial N_{\alpha x}}{\partial x} + \frac{\partial N_{\alpha y}}{\partial y} = I_\alpha \ddot{\phi}_\alpha + J_\alpha \ddot{\theta}_\alpha - c_\alpha \ddot{\phi}_\alpha \]  

(11.b)

\[ \frac{\partial \ddot{q}}{\partial x} + \frac{\partial \ddot{q}}{\partial y} + \left( \frac{\partial}{\partial x} \left( N_{\alpha x} \ddot{w}_\alpha + N_{\alpha y} \ddot{w}_\alpha \right) \right) + \frac{\partial}{\partial y} \left( N_{\alpha x} \ddot{w}_\alpha + N_{\alpha y} \ddot{w}_\alpha \right) \]
\[ + c_i \left( \frac{\partial^3 P_i}{\partial x^3} + 2 \frac{\partial^2 P_i}{\partial x^2 \partial y} + \frac{\partial^3 P_i}{\partial y^3} \right) + q = I_i \dot{w}_i + c_i I_1 \left( \frac{\partial \dot{w}_i}{\partial x} + \frac{\partial \ddot{w}_i}{\partial y} \right) \]

\[ + c_i \left[ I_1 \left( \frac{\partial \ddot{u}_i}{\partial x} + \frac{\partial \ddot{v}_i}{\partial y} \right) + I_4 \left( \frac{\partial \ddot{w}_i}{\partial x} + \frac{\partial \ddot{w}_i}{\partial y} \right) \right] \quad (11.c) \]

\[ \frac{\partial M_{\alpha \alpha}}{\partial x} + \frac{\partial M_{\beta \beta}}{\partial y} - \overline{Q}_\alpha = J_i \ddot{u}_i + k_i \dddot{v}_i - c_i J_i \frac{\partial \ddot{w}_i}{\partial x} \quad (11.d) \]

\[ \frac{\partial M_{\alpha \alpha}}{\partial x} + \frac{\partial M_{\beta \beta}}{\partial y} - \overline{Q}_\beta = J_i \ddot{v}_i + k_i \dddot{v}_i - c_i J_i \frac{\partial \ddot{w}_i}{\partial y} \quad (11.e) \]

Where

\[ I_i = \sum_{j=1}^{i-1} \rho^j \beta^j dz, \quad J_i = J_i - c_i I_{i+2}, \quad K_i = I_i - 2c_i I_i \quad (12) \]

\[ N_{\alpha \alpha}, N_{\beta \beta}, M_{\alpha \alpha}, M_{\beta \beta}, M_{\alpha \beta}, P_{\alpha \alpha}, P_{\beta \beta}, \overline{Q}_\alpha \text{ and } \overline{Q}_\beta \text{ are the stress resultants related} \]

to the strains.

\[ N_{\alpha \alpha} = A_1 \left( \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \right) + A_2 \left( \frac{\partial v_0}{\partial y} + \frac{1}{2} \left( \frac{\partial w_0}{\partial y} \right)^2 \right) \]

\[ - N_{\alpha \alpha}' - N_{\beta \beta}' - q_i \cos \Omega t \quad (13.a) \]

\[ N_{\beta \beta} = A_3 \left( \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \right) + A_2 \left( \frac{\partial v_0}{\partial y} + \frac{1}{2} \left( \frac{\partial w_0}{\partial y} \right)^2 \right) \]

\[ - N_{\alpha \beta}' - N_{\beta \alpha}' - q_i \cos \Omega t \quad (13.b) \]

\[ N_{\alpha \beta} = A_4 \left( \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial u_0}{\partial y} + \frac{\partial u_0}{\partial y} \right) \]

\[ - N_{\alpha \beta}' - N_{\beta \alpha}' \quad (13.c) \]

\[ M_{\alpha \alpha} = \left( D_{11} - c_i F_{11} \right) \frac{\partial \phi_0}{\partial x} + \left( D_{21} - c_i F_{21} \right) \frac{\partial \phi_0}{\partial y} - c_i F_{21} \frac{\partial^2 w_0}{\partial x^2} \]

\[ - c_i F_{21} \frac{\partial^2 w_0}{\partial y^2} - M_{\alpha \alpha}' - M_{\alpha \beta}' \quad (13.d) \]

\[ M_{\alpha \beta} = \left( D_{21} - c_i F_{21} \right) \frac{\partial \phi_0}{\partial x} + \left( D_{22} - c_i F_{22} \right) \frac{\partial \phi_0}{\partial y} - c_i F_{21} \frac{\partial^2 w_0}{\partial x^2} \]

\[ - c_i F_{22} \frac{\partial^2 w_0}{\partial y^2} - M_{\alpha \beta}' - M_{\beta \alpha}' \quad (13.e) \]

\[ M_{\beta \alpha} = \left( D_{12} - c_i F_{12} \right) \frac{\partial \phi_0}{\partial x} + \left( D_{22} - c_i F_{22} \right) \frac{\partial \phi_0}{\partial y} - c_i F_{22} \frac{\partial^2 w_0}{\partial x^2} \]

\[ - c_i F_{22} \frac{\partial^2 w_0}{\partial y^2} - M_{\beta \alpha}' - M_{\beta \beta}' \quad (13.f) \]

\[ P_{\alpha \alpha} = (F_{11} - c_i H_{11}) \frac{\partial \phi_0}{\partial x} + (F_{12} - c_i H_{12}) \frac{\partial \phi_0}{\partial y} - 2c_i F_{12} \frac{\partial^2 w_0}{\partial x^2} \]

\[ P_{\beta \beta} = (F_{12} - c_i H_{12}) \frac{\partial \phi_0}{\partial x} + (F_{22} - c_i H_{22}) \frac{\partial \phi_0}{\partial y} - c_i H_{22} \frac{\partial^2 w_0}{\partial x^2} \]
\[ -c_iH_{z} \frac{\partial^2 w_{a}}{\partial y^2} - P^r_{y} - P^r_{y} 
\]

\[ M_{yy} = (F_{w} - c_iH_{z} \left( \frac{\partial \phi_{y}}{\partial x} + \frac{\partial \phi_{y}}{\partial y} \right) - c_iH_{z} \frac{\partial^2 w_{a}}{\partial x^2} \]

\[ -c_iH_{z} \frac{\partial^2 w_{a}}{\partial y^2} - P^r_{y} - P^r_{y} \]

\[ P_{y} = (P^{\infty} - c_iH_{z} \left( \frac{\partial \phi_{y}}{\partial x} + \frac{\partial \phi_{y}}{\partial y} \right)) - 2c_iH_{z} \frac{\partial^2 w_{a}}{\partial x \partial y} \]

\[ \overline{Q}_{y} = (A_{w} - c_iD_{a}) \phi_{y} + (A_{w} - c_iD_{a}) \frac{\partial w_{y}}{\partial y} \]

\[ \overline{Q}_{y} = (A_{w} - c_iD_{a}) \phi_{y} + (A_{w} - c_iD_{a}) \frac{\partial w_{y}}{\partial x} \]

where

\[ (A_{w}, B_{w}, D_{w}, E_{w}, F_{w}, H_{w}) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_{i}} \overline{Q}_{y}(t, z, z', z'', z; z) \, dt \quad (i, j = 1, 2, 6) \]  \hspace{1cm} (14.a)

\[ (A_{w}, D_{w}, F_{w}) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_{i}} \overline{Q}_{y}(t, z', z) \, dt \quad (i, j = 4, 5) \]  \hspace{1cm} (14.b)

\[ E_{y} = \frac{1}{4} \sum_{i=1}^{n} \overline{Q}_{y}(z_{i}, z' - z) \, dt \quad (i, j = 4, 5) \]  \hspace{1cm} (14.c)

\[ F_{y} = \frac{1}{5} \sum_{i=1}^{n} \overline{Q}_{y}(z_{i}, z' - z) \, dt \quad (i, j = 4, 5) \]  \hspace{1cm} (14.d)

\[ H_{y} = \frac{1}{7} \sum_{i=1}^{n} \overline{Q}_{y}(z_{i}, z' - z) \, dt \quad (i, j = 4, 5) \]  \hspace{1cm} (14.e)

Substitution of the stress resultants equations (13) into the Euler-Lagrange equations in (11) yields the equations of motion in terms of displacements.

\[ A_{w} \frac{\partial^2 v_{w}}{\partial x^2} + A_{w} \frac{\partial^2 v_{w}}{\partial y^2} + (A_{w} + A_{w}) \frac{\partial^2 v_{w}}{\partial x \partial y} + A_{w} \frac{\partial w_{a}}{\partial x} + A_{w} \frac{\partial w_{a}}{\partial y} \]

\[ + (A_{w} + A_{w}) \frac{\partial w_{a}}{\partial x} \frac{\partial^2 w_{a}}{\partial y^2} = I_{w} \ddot{u}_{w} + J_{w} \ddot{v}_{w} - c_iH_{z} \frac{\partial w_{a}}{\partial x} \]  \hspace{1cm} (15.a)

\[ A_{w} \frac{\partial^2 v_{w}}{\partial x^2} + A_{w} \frac{\partial^2 v_{w}}{\partial y^2} + (A_{w} + A_{w}) \frac{\partial^2 v_{w}}{\partial x \partial y} + A_{w} \frac{\partial w_{a}}{\partial x} + A_{w} \frac{\partial w_{a}}{\partial y} \]

\[ + (A_{w} + A_{w}) \frac{\partial w_{a}}{\partial x} \frac{\partial^2 w_{a}}{\partial y^2} = I_{w} \ddot{u}_{w} + J_{w} \ddot{v}_{w} - c_iH_{z} \frac{\partial w_{a}}{\partial y} \]  \hspace{1cm} (15.b)

\[ A_{w} \frac{\partial^2 v_{w}}{\partial x^2} + c_iH_{z} \frac{\partial^2 v_{w}}{\partial y^2} - c_i^2 H_{z} \frac{\partial^2 v_{w}}{\partial x^2 \partial y^2} + A_{w} \left( \frac{\partial w_{a}}{\partial y} \right)^2 \]  \hspace{1cm} (15.c)
We mainly consider nonlinear dynamics of the laminated composite piezoelectric rectangular plates in the first modes of \( u_0 \) and \( v_0 \), the first two modes of
$w_0$ and the first three modes of $\phi_i$ and $\phi_j$. It is our desirable to choose a suitable mode function to the boundary condition for the laminated composite piezoelectric rectangular plates. Thus we write $u_0, v_0, w_0, \phi_x$ and $\phi_y$ in the forms as showed below [19] [20].

\[
\begin{align*}
&u_0 = u_i(t) \cos \frac{\pi x}{2a} \sin \frac{\pi y}{b} \\
v_0 = v_i(t) \cos \frac{\pi y}{2b} \sin \frac{\pi x}{a} \\
w_0 = w_i(t) \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_i(t) \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} \\
&\phi_x = \phi_i(t) \cos \frac{\pi y}{b} \sin \frac{\pi x}{a} + \phi_x(t) \cos \frac{3\pi y}{b} \sin \frac{3\pi x}{a} + \phi_x(t) \cos \frac{3\pi y}{b} \sin \frac{\pi x}{a} \\
&\phi_y = \phi_i(t) \cos \frac{\pi x}{b} \sin \frac{\pi y}{a} \sin \frac{3\pi y}{b} \sin \frac{3\pi x}{a} + \phi_x(t) \cos \frac{3\pi x}{b} \sin \frac{3\pi y}{a} \sin \frac{3\pi x}{a}
\end{align*}
\]

By means of Galerkin method, substituting equations (16) into equations (15) and integrating, we can obtain the expressions of $u_i, v_i, \phi_x, \phi_y, \phi_z, \phi_x$ and $\phi_y$ in the forms of $w_i$ and $w_z$, neglecting all inertia terms in equation (15.a), (15.b), (15.d) and (15.e).

\[
\begin{align*}
u_i &= k_1 w_i^2 + k_2 w_z^2 + k_3 w_i w_z \\
v_i &= k_1 w_i^2 + k_2 w_z^2 + k_3 w_i w_z \\
&\phi_x = k_1 w_x, \phi_y = 0, \phi_z = k_s w_y \\
&\phi_y = k_1 w_y, \phi_x = k_1 w_x, \phi_y = 0
\end{align*}
\]

Where $k_1$ is showed below, and $k_1 \sim k_4$ are similar to $k_i$.

\[
k_1 = 128\pi[(128 - 9\pi^2)A_{25}(A_{36} - A_N)a^2 - a^2 b^2][(64 - 36\pi^2)A_{36}^2 \\
+ 72\pi^2 A_{15} A_{36} b^2 - 64(A_{15} + A_{36})A_{36} - 18\pi^2 A_{15} A_{36} \\
+ (36\pi^2 + 64)A_{25} A_{36}] / 5 [324\pi^2 A_{25} A_{36} a^4 + 324\pi^2 A_{36} A_{36} a^4 \\
+ 324\pi^2 A_{36} a^4 + a^2 b^2][(1296\pi^2 - 4096)A_{36}^2 \\
+ 81\pi^2 A_{25} A_{36} - 4096 A_{36}(A_{36} + A_{36}) - 4096 A_{36} A_{36}] 
\]

(18)
In order to obtain the dimensionless equations, we introduce the transformations of variables and parameters.

\[
\frac{w_1}{a} \rightarrow \frac{w}{a}, \quad \frac{w_2}{a} \rightarrow \frac{w}{a}, \quad x \rightarrow \frac{x}{a}, \quad y \rightarrow \frac{y}{a}, \quad t \rightarrow \frac{t}{T} \quad (19)
\]

Substitution of the equations (16), (17) and (19) into the equations (15c) and integrating, we obtain the equations of motion for the dimensionless as follows:

\[
\begin{align*}
& m_1 \ddot{w}_1 + \omega_n^2 \omega_n^2 w_1 + a \Omega_1 \cos \Omega_1 t + a \cos \Omega_1 t w_1 + a \omega_n^2 \omega_n^2 w_1 + a \omega_n^2 \omega_n^2 w_1 + f \cos \Omega_1 t = 0 \quad (20.a) \\
& m_2 \ddot{w}_2 + \omega_n^2 \omega_n^2 w_2 + b \Omega_2 \cos \Omega_2 t + b \cos \Omega_2 t w_2 + b \omega_n^2 \omega_n^2 w_2 + b \omega_n^2 \omega_n^2 w_2 + f \cos \Omega_2 t = 0 \quad (20.b)
\end{align*}
\]

Parts of the parameters are showed below.

\[
\begin{align*}
M_1 &= \frac{I_a}{T^2} - \frac{c_i J_i k_i \pi}{a T} + \frac{c_i J_i k_i \pi}{a T} + \frac{c_i J_i k_i \pi}{a T} \\
M_2 &= \frac{I_a}{T^2} - \frac{c_i J_i k_i \pi}{a T} + \frac{c_i J_i k_i \pi}{a T} + \frac{c_i J_i k_i \pi}{a T} \\
&= \pi \left( \frac{q_a}{a} + \pi \right) - \frac{\pi \left( \frac{q_a}{a} + \pi \right)}{a^2 + ab^2} \\
&= \frac{\pi}{151200a b^4}(-127575 \pi b^4 A_1 + 42525 \pi^2 a b A_2 + 151200 \pi N_2 a^4 b^4 b^4 + 151200 \pi \pi N_2 a^2 b^4 + 567250 \pi^2 A_4 a^2 b^2 b^2 + 23625 \pi \pi A_4 a^2 b^4 + 374220 a k_a a^2 b^4 + 374220 A_2 k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 - 10240 A_4 k_a a^2 b^4 + 43008 (A_m + A_1) k_a a^2 b^4 - 387072 A_2 k_a a^2 b^4 - 151200 \pi H_1 c_i a^2 b^2 - 151200 \pi H_2 c_i a^2 b^2 - 30240 \pi F_{w_1} c_i a^2 b^2 b^2 + 215040 (A_m + A_1) k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 - 997240 A_1 k_a a^2 b^4 - 151200 \pi H_1 c_i a^2 b^4 + 302400 \pi^2 F_{w_1} c_i a^2 b^4 + 151200 \pi H_2 c_i a^2 b^4 + 151200 \pi H_2 c_i a^2 b^4 + 604800 \pi H_1 c_i a^2 b^4 - 151200 \pi H_2 c_i a^2 b^4) \\
&= \frac{\pi}{151200a b^4}(-127575 \pi b^4 A_1 + 42525 \pi^2 a b A_2 + 151200 \pi N_2 a^4 b^4 b^4 + 151200 \pi \pi N_2 a^2 b^4 + 567250 \pi^2 A_4 a^2 b^2 b^2 + 23625 \pi \pi A_4 a^2 b^4 + 374220 a k_a a^2 b^4 + 374220 A_2 k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 - 10240 A_4 k_a a^2 b^4 + 43008 (A_m + A_1) k_a a^2 b^4 - 387072 A_2 k_a a^2 b^4 - 151200 \pi H_1 c_i a^2 b^2 - 151200 \pi H_2 c_i a^2 b^2 - 30240 \pi F_{w_1} c_i a^2 b^2 b^2 + 215040 (A_m + A_1) k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 - 997240 A_1 k_a a^2 b^4 - 151200 \pi H_1 c_i a^2 b^4 + 302400 \pi^2 F_{w_1} c_i a^2 b^4 + 151200 \pi H_2 c_i a^2 b^4 + 151200 \pi H_2 c_i a^2 b^4 + 604800 \pi H_1 c_i a^2 b^4 - 151200 \pi H_2 c_i a^2 b^4) \\
&= \frac{\pi}{151200a b^4}(-127575 \pi b^4 A_1 + 42525 \pi^2 a b A_2 + 151200 \pi N_2 a^4 b^4 b^4 + 151200 \pi \pi N_2 a^2 b^4 + 567250 \pi^2 A_4 a^2 b^2 b^2 + 23625 \pi \pi A_4 a^2 b^4 + 374220 a k_a a^2 b^4 + 374220 A_2 k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 - 10240 A_4 k_a a^2 b^4 + 43008 (A_m + A_1) k_a a^2 b^4 - 387072 A_2 k_a a^2 b^4 - 151200 \pi H_1 c_i a^2 b^2 - 151200 \pi H_2 c_i a^2 b^2 - 30240 \pi F_{w_1} c_i a^2 b^2 b^2 + 215040 (A_m + A_1) k_a a^2 b^4 + 215040 (A_m + A_1) k_a a^2 b^4 - 997240 A_1 k_a a^2 b^4 - 151200 \pi H_1 c_i a^2 b^4 + 302400 \pi^2 F_{w_1} c_i a^2 b^4 + 151200 \pi H_2 c_i a^2 b^4 + 151200 \pi H_2 c_i a^2 b^4 + 604800 \pi H_1 c_i a^2 b^4 - 151200 \pi H_2 c_i a^2 b^4)
\end{align*}
\]
\[ 3. \, PERTURBATION \, ANALYSIS \]

To obtain the averaged equation of (20), we use the method of multiple scales \[18\] in the following form:

\[ w_1(t, \varepsilon) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + \cdots \] \hspace{1cm} (25)

\[ w_2(t, \varepsilon) = y_0(T_0, T_1) + \varepsilon y_1(T_0, T_1) + \cdots \] \hspace{1cm} (26)

where \( T_0 = t \), \( T_1 = \omega t \). Then we have the differential operators:

\[ \frac{D}{dt} = \frac{\partial}{\partial T_0} + \frac{\partial}{\partial T_1} + \cdots = D_0 + \varepsilon D_1 + \cdots \] \hspace{1cm} (27)

\[ \frac{D^2}{dt^2} = (D_0 + \varepsilon D_1 + \cdots)^2 = D_0^2 + 2\varepsilon D_0 D_1 + \cdots \] \hspace{1cm} (28)

Where \( D_0 = \frac{\partial}{\partial T_0}, D_1 = \frac{\partial}{\partial T_1} \).

We only study the case of primary parametric resonance and 1:2 internal resonance. In this resonant case these are the following relations:

\[ \omega_1^2 = \frac{\omega^2}{4} + \varepsilon \sigma_1, \quad \omega_2^2 = \omega^2 + \varepsilon \sigma_1, \quad \text{and} \quad \Omega_1 = \Omega_2 = \Omega_3 = \Omega_4 = \omega \] \hspace{1cm} (29)

Substituting equations (25)-(29) into equations (20) and balancing the coefficients of like power of \( \varepsilon \) on the left-hand and right-hand sides of the equations, the differential equations are obtained as follows:

order \( \varepsilon^0 \)

\[ D_0^2 x_0 + \frac{\omega^2}{4} x_0 = 0 \] \hspace{1cm} (30)

\[ D_0^2 y_0 + \omega^2 y_0 = 0 \] \hspace{1cm} (31)

order \( \varepsilon \)

\[ D_0^2 x_1 + \frac{\omega^2}{4} x_1 = a_1 D_0 x_0 + (a_2 + a_3) \cos(\omega t) x_0 - \sigma_1 x_0 + a_4 y_0 \]
\[ + a_5 x_0 y_0 + a_6 x_0^2 y_0 + a_7 x_0 y_0^2 + f \cos(\omega t) + 2D_0 D_1 x_0 \] \hspace{1cm} (30)

\[ D_0^2 y_1 + \frac{\omega^2}{4} y_1 = b_1 D_0 y_0 + (b_2 + b_3) \cos(\omega t) y_0 - \sigma_1 y_0 + b_4 y_0^2 x_0 \]
The solutions in the complex form of equations (30) and (31) can be found as

\[ x_0 = A(T)e^{i\omega_0 t} + \overline{A}(T)e^{-i\omega_0 t} \]  
\[ y_0 = B(T)e^{i\omega_0} + \overline{B}(T)e^{-i\omega_0} \]

Where \( \overline{A} \) and \( \overline{A} \) are the complex conjugate of \( A \) and \( B \), respectively. Substituting equation (32) and (33) into equations (30) and (31) yields

\[ D_0^2 x_i + \frac{\sigma}{2} \omega^2 x_i = \left[ -\sigma, A + \frac{\sigma}{2}(a_0 + a_r + a_\omega), \overline{A} - i\omega D, A + \frac{\sigma}{2}ia_0, \overline{A} \right] + 2a_r AB \overline{B} + 3a_0 A \overline{\overline{A}} e^{i\omega_0 t} + cc + NST \]  
\[ D_0^2 y_i + \omega^2 y_i = \left[ -\sigma, B - 2i\omega D, B + ib_0, \omega B + \frac{\sigma}{2} f, \right] + 3b_0 B \overline{\overline{B}} e^{i\omega_0} + cc + NST \]

where \( cc \) represents the parts of the complex conjugate of the function on the right-hand side of equations (34) and (35), and \( NST \) represents the terms that do not produce secular terms. Eliminating the terms that produce secular terms from equations (34) and (35) yields

\[ D_0^2 A + \frac{1}{2} a_r A + \frac{1}{2} i\sigma, A - i\sigma, A - \frac{3}{4} i\sigma, A - \frac{1}{4} i\sigma, A \]  
\[ D_0^2 B = \frac{1}{2} b_0 B + \frac{1}{4} i\sigma, B - \frac{1}{2} i\sigma, B - \frac{3}{4} i\sigma, B - \frac{1}{8} i f \]

The functions \( A \) and \( B \) may be expressed in the complex form

\[ A = x_1 + ix_2, \quad B = x_1 + ix_2 \]

Substituting equation (38) into equations (36) and (37), the averaged equations in the complex form are obtained as follows:

\[ D_0^2 x_1 = \frac{1}{2} a_r x_1 - \frac{1}{2} i\sigma, x_2 - \frac{3}{4} (a_0 + a_r + a_\omega)x_2 \]
\[ + a_r x_1 (x_1^2 + x_2^2) + \frac{3}{2} a_r x_1 (x_1^2 + x_2^2) \]  
\[ D_0^2 x_2 = \frac{1}{2} a_r x_1 + \frac{1}{2} i\sigma, x_1 - \frac{3}{4} (a_0 + a_r + a_\omega)x_1 \]
5. Numerical simulation of chaos

The global perturbation method developed by Kovacic and Wiggins can only be used to analyze the autonomous systems, but cannot be used to analyze the non-autonomous systems. So we choose the averaged equations (39) – (42) to do the numerical simulations. The software of VC is chosen to find a numerical solution of ordinary differential equations, using a fourth-fifth order Runge-Kutta method. Considered the averaged equations (39) – (42), firstly, the case for \( a_1 = 400.3 \) and \( f_1 = 322.3 \) is numerically studied. The chosen other parameters and initial conditions are \( a_1 = 0.001 \), \( b_1 = 0.001 \), \( \sigma_1 = 0.001 \), \( \sigma_2 = 4.35 \), \( a_2 = -12.9 \), \( a_3 = -13.1 \), \( b_2 = 8.6 \) and \( b_3 = -6.5 \). The quasi-periodic response of the first mode and the chaotic response of the second mode are shown in Fig.2. Fig.2(a) and 2(c) respectively represent the phase portraits on the planes \((x_1, x_t)\) and \((x_2, x_t)\). Fig. 2(b) and 2(d) respectively indicate the waveforms on the planes \((t, x_1)\) and \((t, x_2)\). Fig. 2(e) and 2(f) represent Poincare map on plane \((x_1, x_t)\) and three-dimensional phase portrait in the space \((x_1, x_2, x_t)\), respectively.
When the excitation is changed to $a_x = 132.3$ and $f_z = 252.3$, the chaotic motion of the laminated composite piezoelectric rectangular plates are shown in Fig. 3. The other parameters are the same as those in Figure 2.
When the excitation is changed to $a_2 = 102.3$ and $f_2 = 62.3$, the chaotic motion of the laminated composite piezoelectric rectangular plates are shown in Fig. 4. The other parameters are the same as those in Figure 2.
Fig. 4. The chaotic motion of the laminated composite piezoelectric rectangular plates
6 Conclusions

We considered the laminated composite piezoelectric rectangular plates supported along edges. The Reddy's third-order shear deformation plate theory was chosen to describe Laminated Composite Rectangular Plates. The effects of piezoelectric property of the Laminated Composite Rectangular Plates are considered simultaneously. The governing differential equations were obtained using the dynamic version of the principle of virtual displacements. And, two coupling nonlinear transverse oscillations, including in-planar and out-planar, are considered for the Laminated Composite Piezoelectric Plates. Then, the method of multiple scales was applied directly to obtain governing equation of motion. We obtained a four-dimensional averaged equation under the case of 1:2 internal resonances and primary parametric resonance of the first order mode. In the situation investigated in this paper, the forcing excitation is considered to be a controlling parameter to control the dynamic response of the Laminated Composite Piezoelectric Plates. Finally, numerical method is used to investigate chaotic motions of the Laminated Composite Piezoelectric Plates. The results of numerical simulation demonstrate that there exist the periodic and chaotic motions of the Laminated Composite Piezoelectric Plates. The phase portrait, waveform and power spectrum are plotted to show that the chaotic motions occur in the planar nonlinear vibration of the Laminated Composite Piezoelectric Plates under certain conditions. The numerical results show the existence of chaotic motion in averaged equations, the chaotic responses are sensitive to initial conditions especially sensitive to forcing loads and the parametric excitation.

References


Appendix

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